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Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point

Abstract

The focus of this paper is the problem of foot rotation in biped robots during the single-support phase. Foot rotation is an indication of postural instability, which should be carefully treated in a dynamically stable walk and avoided altogether in a statically stable walk.

We introduce the foot-rotation indicator (FRI) point, which is a point on the foot/ground-contact surface where the net ground-reaction force would have to act to keep the foot stationary. To ensure no foot rotation, the FRI point must remain within the convex hull of the foot-support area.

In contrast with the ground projection of the center of mass (GCoM), which is a static criterion, the FRI point incorporates robot dynamics. As opposed to the center of pressure (CoP)—better known as the zero-moment point (ZMP) in the robotics literature—which may not leave the support area, the FRI point may leave the area. In fact, the position of the FRI point outside the footprint indicates the direction of the impending rotation and the magnitude of rotational moment acting on the foot. Owing to these important properties, the FRI point helps not only to monitor the state of postural stability of a biped robot during the entire gait cycle, but indicates the severity of instability of the gait as well. In response to a recent need, the paper also resolves the misconceptions surrounding the CoP/ZMP equivalence.

KEY WORDS—biped robot, foot-rotation indicator (FRI) point, zero-moment point (ZMP), foot rotation, postural stability, stability margin

1. Motivation

The problem of gait planning for biped robots is fundamentally different from the path planning for traditional fixed-base manipulator arms, as is succinctly pointed out by Vukobratovic, et al. (1990). A biped robot may be viewed as a ballistic mechanism which intermittently interacts with its environment—the ground—through its feet. The foot/ground "joint" is *unilateral*, since attractive forces are not present, and *underactuated*, since control inputs are absent. Formally speaking, unilaterality and underactuation are the inherent characteristics of legged locomotion, and at the same time, are the root causes behind their postural instability and fall. A loss of postural stability may have potentially serious consequences, and this calls for its thorough analysis to better predict and eliminate the possibility of fall.

Postural balance and stance-foot equilibrium are profoundly intertwined. A biped-robot gait is said to be statically stable (Shih 1996) and a human posture is said to be balanced (Patla, Frank, and Winter 1990) if the gravity line from its center of mass (or GCoM: Ground projection of the Center of Mass) falls within the convex hull of the foot-support area (henceforth called the support polygon). It is worth noting that a human being can almost always regain the upright posture as long as the feet are securely posed on the ground. The exit of the GCoM from the support polygon is equivalent to the presence of an uncompensated moment on the foot, which causes it to rotate about a point on the polygon boundary.

Rotational equilibrium of the foot is therefore an important criterion for the evaluation and control of gait and postural stability in legged robots. Indeed, foot rotation has been noted to reflect a loss of balance and an eventual fall in monopods (Lee and Raibert 1991) and bipeds (Arakawa and Fukuda 1997)—two classes of legged robots most prone to instabilities. The exit of the GCoM from the support polygon is considered to be the determining factor of stability in the study of human posture as well (Patla, Frank, and Winter 1990). Among the several ways in which the static equilibrium of the robot foot may be disturbed, such as pure sliding, pure rotation about a boundary point, composite sliding and rotation, and even a complete detachment, this paper addresses the initiation of pure foot rotation.

Although the position of the GCoM is sufficient to determine the occurrence of foot rotation in a stationary robot, it is not so for a robot in motion. Instead, it is the location of the

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foot-rotation indicator (FRI) point, which we introduce in this paper, that indicates the existence of an unbalanced torque on the foot. The FRI point is a point on the foot/ground surface, within or outside the support polygon, where the net ground-reaction force *would have to act* to keep the foot stationary. The farther away this point is from the support boundary, the larger the unbalanced moment, and the greater the instability. To ensure no foot rotation, the FRI point must remain within the support polygon, regardless of the GCoM position. The FRI point is a dynamics-based criterion, and reduces to the GCoM position for a stationary robot.

We emphasize that the FRI point is distinctly different from the center of pressure (CoP), better known as the zero-moment point or ZM in the robotics literature (Arakawa and Fukuda 1997; Hemami and Golliday 1977; Hirai, et al. 1998; Li, Takanishi, and Kato 1993; Shih 1996; Shih et al. 1990; Takanishi, et al. 1985; Vukobratovic, et al. 1990), and frequently used in gait planning for biped robots. The CoP is a point on the foot/ground surface where the net ground-reaction force *actually acts*. Regardless of the state of stability of the robot, the CoP may never leave the support polygon, whereas the FRI point does so whenever there is an unbalanced torque on the foot. In fact, the distance of the FRI point from the support polygon is an indication of the severity of this unbalanced torque, and may be exploited during the planning stage.

This paper makes two contributions. The main contribution is the introduction of the FRI point, which may be employed as a useful tool for gait planning in biped and other legged robots, as well as for the postural stability assessment in the human. The second contribution is in response to our discussions with other researchers regarding the misconceptions surrounding the CoP/ZMP equivalence. We review the basics of both concepts, and show that they are identical.

1.1. Some Comments

Although our work is inspired by the analogy between the biped robot gait and human locomotion, we do not explicitly investigate human locomotion in this paper. The discussion refers uniquely to robots, with the implicit understanding that the developed concepts may be extended to the study of human locomotion.

The FRI-point concept may be applied to other multilegged robots. We limit ourselves to biped robots, because postural stability and fall-related issues are especially important to statically unstable robots. Our main focus is the single-support stage of the locomotion cycle, during which only one foot, called the support foot, is in contact with the ground, while the other leg swings forward. In the typical human gait, the single-support stage occupies about 80% of the entire gait cycle (Winter, Ruder, and MacKinnon 1990).

We address the mechanics of foot rotation, and do not concern ourselves with the formulation or implementation of any control law. However, since the real interest in this area results from control problems, a brief description of the control issues is included for completeness (in Section 5). Please note that whenever the context permits, we loosely use "force" to mean "force/torque."

2. FRI Point of a General 3-D Biped Robot

To formally introduce the FRI point, we first treat the entire biped robot—a general n-segment extended rigid-body kinematic chain (see the sketch in Fig. 1 left)—as a system, and determine its response to external force/torque. We may employ Newton's or d'Alembert's principle for this purpose. The external forces acting on the robot are the resultant ground-reaction force/torques, $\boldsymbol{R}$ and $\boldsymbol{M}$, acting at the CoP (denoted by P; see Fig. 1, right), and gravity. The equation for rotational dynamic equilibrium[1] is obtained by noting that the sum of the external moments on the robot, computed either at its GCoM or at *any* stationary reference point, is equal to the sum of the rates of change of angular momentum of the individual segments about the same point. Taking moments at the origin O, we have

$$\begin{aligned} &\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \sum \boldsymbol{OG}_i \times m_i \boldsymbol{g} \\ &\quad = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{OG}_i \times m_i \boldsymbol{a}_i, \end{aligned} \tag{1}$$

where m_i is the mass, G_i is the CoM location, $\boldsymbol{a}_i$ is the CoM linear acceleration, and $\boldsymbol{H}_{Gi}$ is the angular momentum about CoM, of the ith segment.

An important aspect of our approach is to treat the stance foot as the focus of attention of our analysis. Indeed, as the only robot segment interacting with the ground, the stance foot is a "special" segment subjected to joint forces, gravity forces, and the ground-reaction forces. Viewing from the stance foot, the dynamics of the rest of the robot may be completely represented by the ankle force/torque $-\boldsymbol{R}_1$ and $-\boldsymbol{\tau}_1$ (negative signs are for convention). Figure 1 (right) artificially disconnects the support foot from the shank to clearly show the forces in action at that joint. The dynamic equilibrium equation of the foot (segment 1) is

$$\begin{aligned} &\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 \\ &\quad - \boldsymbol{OO}_1 \times \boldsymbol{R}_1 = \dot{\boldsymbol{H}}_{G1} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{a}_1. \end{aligned} \tag{2}$$

The equations for *static* equilibrium of the foot are obtained by setting the dynamic terms (in the right-hand side) in eq. (2) to zero:

$$\boldsymbol{M} + \boldsymbol{OP} \times \boldsymbol{R} + \boldsymbol{OG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{OO}_1 \times \boldsymbol{R}_1 = \boldsymbol{0}. \tag{3}$$

Recall that to derive eq. (3) we could compute the moments at any other stationary reference point. Out of these, the CoP

1. We deal with rotational equilibrium only, and do not discuss translational equilibrium (sliding), assuming that the foot/ground friction is sufficiently large to prevent it.

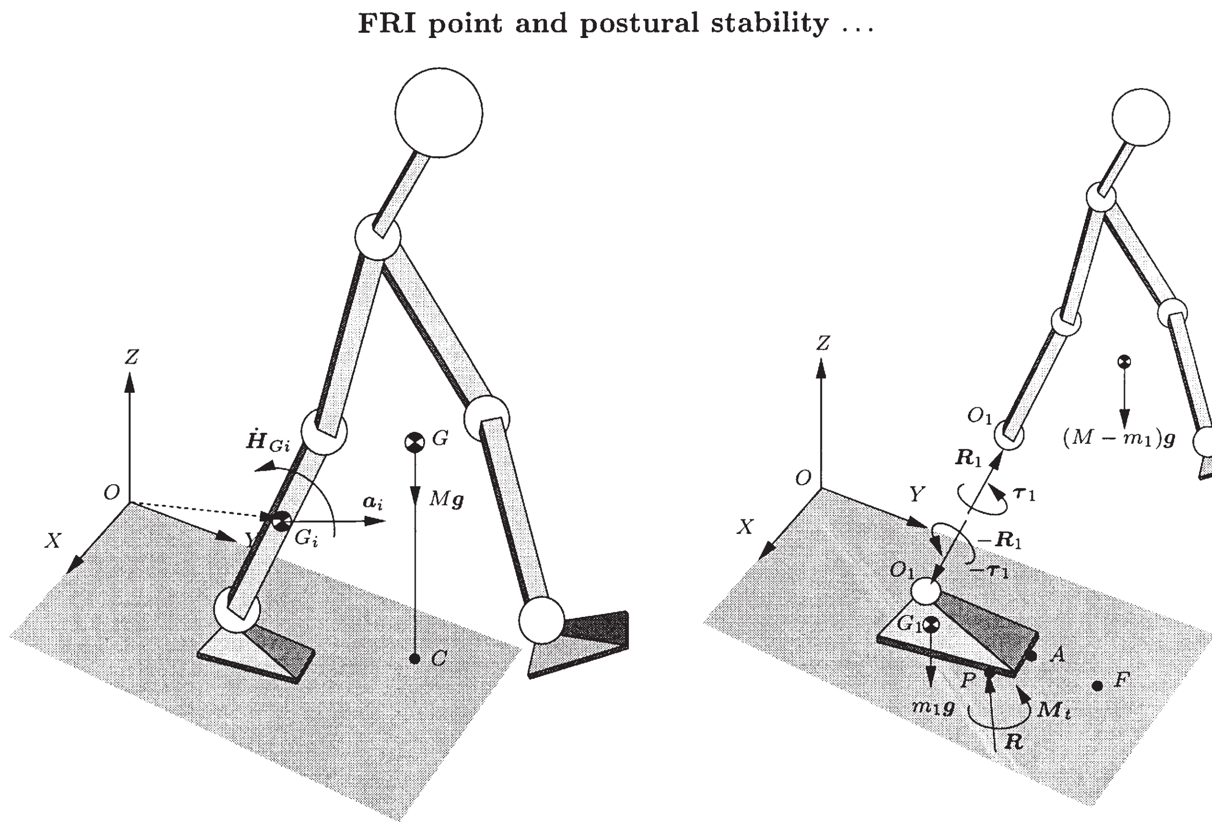

Fig. 1. The sketch of a 3-D extended rigid-body biped robot (left), and a view with its support foot artificially disconnected from the shank to show the intervening forces (right). The CoP, GCoM, and the FRI point are denoted by P, C, and F, respectively.

represents a special point where eq. (3) reduces to a simpler form,

$$\boldsymbol{M} + \boldsymbol{PG}_1 \times m_1\boldsymbol{g} - \boldsymbol{\tau}_1 - \boldsymbol{PO}_1 \times \boldsymbol{R}_1 = \boldsymbol{0}. \tag{4}$$

Considering only the tangential (XY) vector components of eq. (4), we may write

$$\left(\boldsymbol{\tau}_1 + \boldsymbol{PO}_1 \times \boldsymbol{R}_1 - \boldsymbol{PG}_1 \times m_1\boldsymbol{g}\right)_t = \boldsymbol{0}, \tag{5}$$

where the subscript t implies the tangential components. Since $\boldsymbol{M}$ is tangential to the foot/ground surface, its vector direction is normal to that surface and does not contribute to this equation.[2]

In the presence of an unbalanced torque on the foot, eq. (5) is not satisfied for any point within the support polygon. One may, however, still find a point F outside the support boundary that satisfies eq. (4); i.e.,

$$\left(\boldsymbol{\tau}_1 + \boldsymbol{FO}_1 \times \boldsymbol{R}_1 - \boldsymbol{FG}_1 \times m_1\boldsymbol{g}\right)_t = \boldsymbol{0}. \tag{6}$$

The point F is called the FRI point, and is defined as *the point on the foot/ground contact surface, within or outside the convex hull of the foot-support area, at which the resultant moment of the force/torque impressed on the foot is normal to the surface.* By "impressed force/torque," we mean the force and torque at the ankle joint, other external forces, plus the weight of the foot, and not the ground-reaction forces. Following the work of Banach (1951), we may identify the impressed forces as the *acting forces*, in contrast to the reaction forces from the ground, which are the *constrain forces*. An intuitive understanding of the FRI point is obtained by setting $\boldsymbol{\tau}_1 = \boldsymbol{0}$, $m_1 = 0$ in eq. (6). In this case, F is simply the point on the ground where the line of action of $\boldsymbol{R}_1$ penetrates, as shown in Figure 2. The case of the unactuated ankle joint was considered by Lee and Raibert (1991) to analyze the hoof rotation in a monopod.

It is important to note that the location of the ankle joint and the geometry of the support-polygon boundary are the only

2. We ignore foot rotation about the ground normal, as it does not contribute to a balance loss.

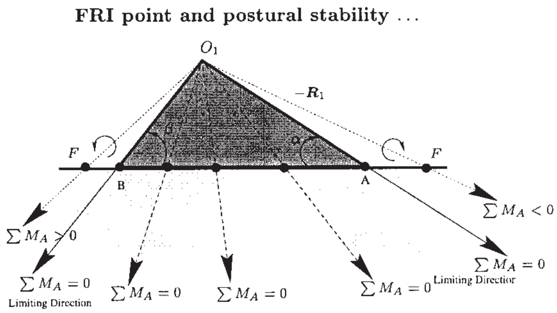

Fig. 2. Condition for foot rotation when $\boldsymbol{\tau}_1 = 0$. The figure sketches different lines of action of the force $\boldsymbol{R}_1$ applied on the robot foot by the rest of the robot at the ankle joint O_1. If the line of action of a force intersects the ground beyond the footprint, there is a net moment applied on the foot and the foot rotates. Otherwise, the ankle joint forces may be supported by the foot/ground interaction forces, and the foot maintains static equilibrium in its stationary upright configuration.

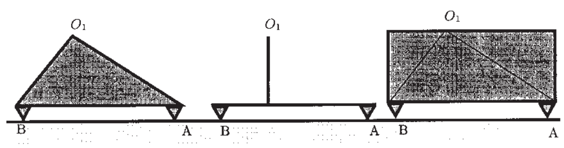

Fig. 3. The locations of key points—the ankle-joint location (O_1) and the support-polygon boundary (A and B)—and not its overall geometry are relevant for the behavior of the foot. The three examples of the robot foot shown in the figure have identical behavior, although their geometries are very different.

important features of the foot that are relevant in our discussion. The actual physical shape of the foot is not important. See Figure 3 for a graphical illustration of this fact.

Explicit expressions for the coordinates of F, $\boldsymbol{OF}(OF_x, OF_y, OF_z = 0)$, are obtained by computing the dynamics of the robot *minus the foot* at F,

$$\begin{aligned}\boldsymbol{\tau}_1 + \boldsymbol{FO}_1 \times \boldsymbol{R}_1 + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i \boldsymbol{g} \\ = \sum_{i=2}^{n} \dot{\mathbf{H}}_{Gi} + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i \boldsymbol{a}_i.\end{aligned} \tag{7}$$

Using eq. (6) and considering only the tangential components,

$$\left(\boldsymbol{FG}_1 \times m_1 \boldsymbol{g} + \sum_{i=2}^{n} \boldsymbol{FG}_i \times m_i(\boldsymbol{g} - \boldsymbol{a}_i)\right)_t = \left(\sum_{i=2}^{n} \dot{\boldsymbol{H}}_{Gi}\right)_t. \tag{8}$$

Noting $\boldsymbol{FG}_i = \boldsymbol{FO} + \boldsymbol{OG}_i$ and $\boldsymbol{OF} = -\boldsymbol{FO}$, eq. (8) may be rewritten as

$$\begin{aligned}&\left(\sum_{i=2}^{n} \boldsymbol{OF} \times m_i(\boldsymbol{a}_i - \boldsymbol{g}) - \boldsymbol{OF} \times m_1 \boldsymbol{g}\right)_t \\ &= \left(-\boldsymbol{OG}_1 \times m_1 \boldsymbol{g} + \sum_{i=2}^{n} \dot{\boldsymbol{H}}_{Gi} + \sum_{i=2}^{n} \boldsymbol{OG}_i \times m_i(\boldsymbol{a}_i - \boldsymbol{g})\right)_t.\end{aligned} \tag{9}$$

Carrying out the operation, we may finally obtain

$$OF_x = \frac{m_1 OG_{1y} g + \sum_{i=2}^{n} m_i OG_{iy}(a_{iz} + g)}{m_1 g + \sum_{i=2}^{n} m_i(a_{iz} + g)} - \frac{\sum_{i=2}^{n} m_i OG_{iz} a_{iy} + \sum_{i=2}^{n} \dot{H}_{Gix}}{m_1 g + \sum_{i=2}^{n} m_i(a_{iz} + g)}, \tag{10}$$

$$OF_y = \frac{m_1 OG_{1x} g + \sum_{i=2}^{n} m_i OG_{ix}(a_{iz} + g)}{m_1 g + \sum_{i=2}^{n} m_i(a_{iz} + g)} - \frac{\sum_{i=2}^{n} m_i OG_{iz} a_{ix} - \sum_{i=2}^{n} \dot{H}_{Giy}}{m_1 g + \sum_{i=2}^{n} m_i(a_{iz} + g)}. \tag{11}$$

2.1. Properties of the FRI Point

Some useful properties of the FRI point which may be exploited in gait planning include the following:

1. The FRI point indicates the *occurrence* of foot rotation, as already described.
2. The location of the FRI point indicates the *magnitude* of the unbalanced moment on the foot. The total moment M_A^I due to the impressed forces about a point A on the support-polygon boundary (Fig. 1, right) is

$$M_A^I = \boldsymbol{AF} \times (m_1 \boldsymbol{g} - \boldsymbol{R}_1), \tag{12}$$

which is proportional to the distance between A and F. If F is situated inside the support polygon, M_A^I is counteracted by the moment due to $\boldsymbol{R}$ and is precisely compensated; see Figure 4 (left) for a planar example. Otherwise, M_A^I is the uncompensated moment that causes the foot to rotate; see Figure 4 (right).

3. The FRI point indicates the *direction* of foot rotation. This we derive from eq. (12), assuming that $m_1 \boldsymbol{g} - \boldsymbol{R}_1$ is directed downward.

4. The FRI point indicates the *stability margin* of the robot. The stability margin of a robot against foot rotation may be quantified by the minimum distance of the support-polygon boundary from the current location of the FRI point within the footprint. Conversely, when the FRI point is outside the footprint, this minimum distance is a measure of instability of the robot. An imminent foot rotation will be indicated by a motion of the FRI point toward the support-polygon boundary.

3. The CoP (ZMP), GCoM, and FRI Point Compared

In this section, we compare and contrast the three quantities: the CoP, the GCoM, and the FRI point. The CoP and GCoM are used both in the robotics literature as well as in biomechanics, and are often a source of misconception and confusion. We will pay particular attention to the concept of the ZMP, and show that it is identical to the CoP. We show that the FRI point better reflects postural instability in a dynamic situation compared with the CoP and the GCoM.

3.1. The CoP Reviewed

Although the concept of "CoP" most likely originated in the field of fluid mechanics, it is frequently used in the study of gait and postural balance. The CoP is defined as *the point on the ground where the resultant of the ground-reaction force acts.*

As shown in Figure 5, two types of interaction forces act on the foot at the foot/ground interface. They are the normal forces $\boldsymbol{f}_{ni}$, always directed upward (Fig. 5, left) and the frictional tangential forces $\boldsymbol{f}_{ti}$ (Fig. 5, center). The CoP may be defined as the point P where the resultant $\boldsymbol{R}_n = \sum \boldsymbol{f}_{ni}$ acts. With respect to a coordinate origin O, $\boldsymbol{OP} = \dfrac{\sum \boldsymbol{q}_i f_{ni}}{\sum f_{ni}}$, where $\boldsymbol{q}_i$ is the vector to the point of action of force $\boldsymbol{f}_i$ and f_i is the magnitude of $\boldsymbol{f}_i$.

The unilaterality of the foot/ground constraint is a key feature of legged locomotion. This means that $\boldsymbol{f}_{ni} \geq 0$, which translates to the fact that P must lie within the support polygon. The resultant of the tangential forces may be represented at P by a force $\boldsymbol{R}_t = \sum \boldsymbol{f}_{ti}$ and a moment $\boldsymbol{M} = \sum \boldsymbol{r}_i \times \boldsymbol{f}_{ti}$, where $\boldsymbol{r}_i$ is the vector from P to the point of application of $\sum \boldsymbol{f}_{ti}$.

The complete picture is shown in Figure 5, right. The stance foot of the biped robot is subjected to a resultant ground- reaction force $\boldsymbol{R} = \boldsymbol{R}_n + \boldsymbol{R}_t$ and a ground-reaction moment $\boldsymbol{M}$. An analysis with a continuous distribution of ground-reaction forces was performed earlier (Coussi and Bessonet 1995; Espiau 1998). We point out that contrary to what appeared in Shih's work (1996), $\boldsymbol{R}$, and not $\boldsymbol{R}_n$, is the total ground-reaction force. Please note that the CoP is identical to what has been termed the "center of the actual ground-reaction force" (C-ATGRF) in a recent paper (Hirai, et al. 1998).

3.2. The Zero-Moment Point (ZMP)

The concept of the ZMP which we demonstrate to be identical to the CoP is known to have originally been introduced in 1969 (Vukobratovic and Juricic 1969). Since then, it has been frequently used in biped robot control as a criterion of postural stability (Arakawa and Fukuda 1997; Hemami and Golliday 1977; Hirai et al. 1998; Takanishi and Kato 1993; Shih 1996; Shih et al. 1990; Takanishi, et al. 1985; Vukobratovic et al. 1990). Reference is often made to the *ZMP condition* (Arakawa and Fukuda 1997), or the *ZMP stability criterion* (Li, Takanishi, and Kato 1993), which states that the ZMP of a biped robot must be constrained within the convex hull of the foot-support area to ensure the stability of the foot/ground contact (Arakawa and Fukuda 1997); the walk stability without falling down (Arakawa and Fukuda 1997); the dynamic stability of locomotion (Shih et al. 1990; Shih 1996); and the physical admissibility and realizability of the gait (Shih 1996). Unfortunately, these terminologies are not all equivalent, and the physical implications of some of them are not entirely clear.

A similar problem is encountered with the different definitions of ZMP, which perhaps due to lack of rigor, are not always clearly understandable. This has created confusion in the research community. Discussions with other researchers have convinced us that in view of the significantly increased interest in biped-robot research in recent times, it is necessary to review and clarify the physics behind the concept of ZMP and remove the existing misconceptions. Instead of attempting to redefine the ZMP, we reproduce some of the definitions that are correct (being all equivalent) and easy to understand:

Definiton 1 (Hemami and Golliday 1977) The ZMP is the point where the vertical reaction force intersects the ground.

Definition 2 (Takanishi, et al. 1985) The ZMP is the point on the ground where the total moment generated due to gravity and inertia equals zero.

Definition 3 (Arakawa and Fukuda 1997) The ZMP is the point on the floor at which the moment $\boldsymbol{T} : (T_x, T_y, T_z)$ generated by the reaction force and the reaction torque satisfies $T_x = 0$, and $T_y = 0$.

Definition 4 (Hirai, et al. 1998) The point on the ground at which the moment of the total inertia force (which the authors previously define as the combination of inertia force and gravity force) becomes zero is called the ZMP.

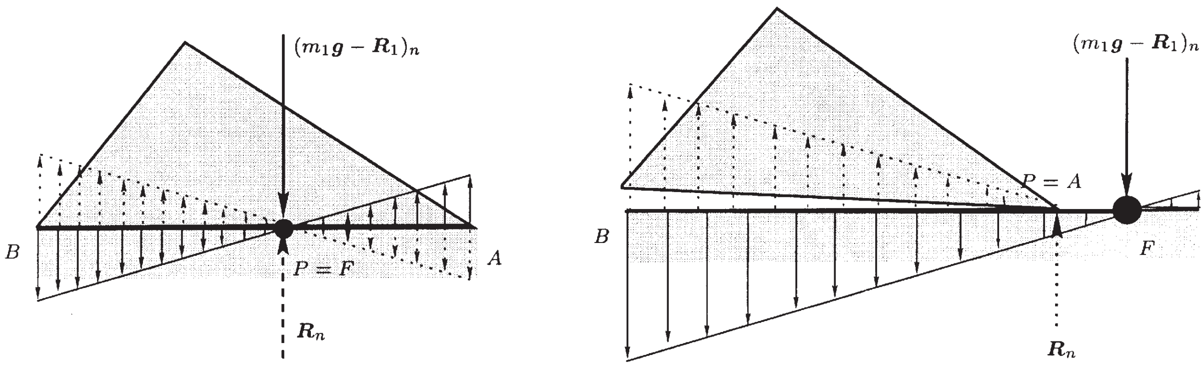

Fig. 4. The magnitude of the moment experienced by a point on the support boundary is linearly proportional to the distance of this point from the FRI point. The magnitudes of the moments at different points are shown by the length of the arrows. Clockwise (i.e., negative) moments are shown by upward-pointed arrows, and counterclockwise (i.e., positive) moments are shown by downward-pointed arrows. In the left image the moments are precisely compensated, whereas in the right image they are not. The subscript n denotes the normal component of a force.

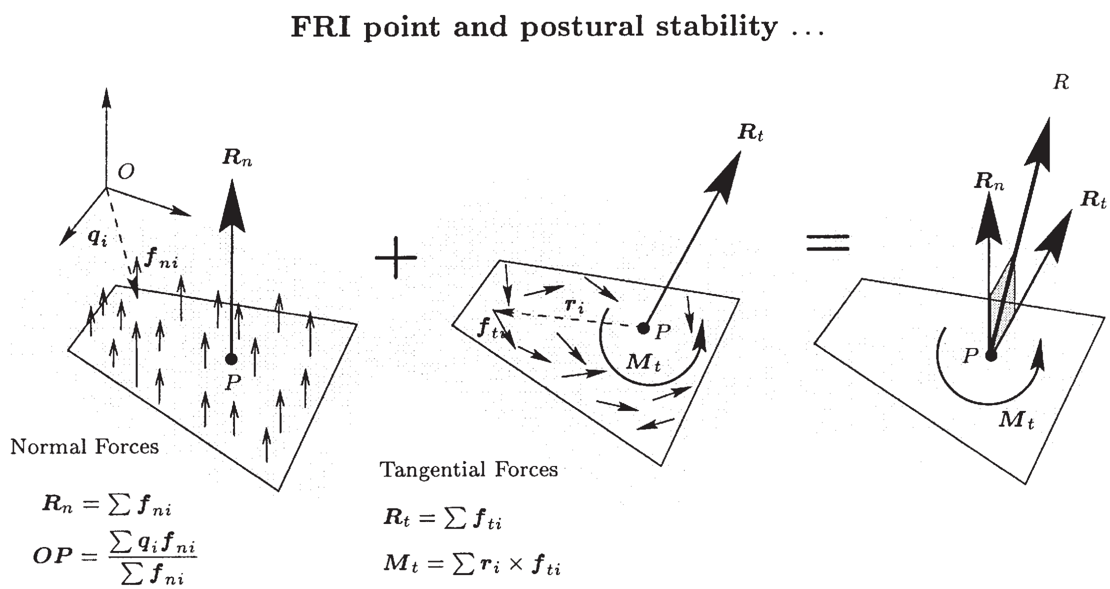

Fig. 5. An analysis of the CoP. In the foot/ground interface, we have the normal forces (left) and the frictional tangential forces (center). The CoP is the point (P) where the resultant $\boldsymbol{R}_n$ of the normal forces acts. At the CoP, the tangential forces may be represented by a resultant force $\boldsymbol{R}_t$ and a moment $\boldsymbol{M}$. The ground-reaction force is $\boldsymbol{R} = \boldsymbol{R}_n + \boldsymbol{R}_t$.

The term *zero*-moment point is a misnomer, since in general only two of the three moment components are zero (Coussi and Bessonet 1995). This raises a question about the necessity of introducing a new name for an already well-known concept, the CoP.

3.3. CoP = ZMP

Definitions 1 and 3 for the ZMP immediately correspond to the definition of the CoP as described in Section 3.1. It is also possible to show that the CoP is the point where the resultant moment generated by the inertia and gravity forces is tangential to the surface (Definitions 2 and 4). To prove this, let us first assume that this latter point, which we call D, is distinct from the CoP. The dynamic equilibrium equation computed at D takes the form

$$\begin{aligned} \boldsymbol{M} + \boldsymbol{DP} \times \boldsymbol{R} + \sum \boldsymbol{DG}_i \times m_i \boldsymbol{g} \\ = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i \boldsymbol{a}_i, \end{aligned} \tag{13}$$

whereas by definition, D satisfies

$$\left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i (\boldsymbol{a}_i - \boldsymbol{g}) \right)_t = \boldsymbol{0}. \tag{14}$$

Comparing eqs. 13 and 14, $(\boldsymbol{DP} \times \boldsymbol{R})_t = \boldsymbol{0}$. However, since $\boldsymbol{R} \neq \boldsymbol{0}$ and $\boldsymbol{DP} \nparallel \boldsymbol{R}$ in general, this is possible only if $\boldsymbol{DP} = \boldsymbol{0}$ or the points D and P are coincident. Other approaches have led to identical conclusions (Coussi and Bessonet 1995; Espiau 1998).

Rewriting eq. (13) as

$$\left(\boldsymbol{DP} \times \boldsymbol{R} \right)_t = \left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{DG}_i \times m_i (\boldsymbol{a}_i - \boldsymbol{g}) \right)_t \tag{15}$$

gives us a clearer picture of the equivalence of CoP and ZMP. Whereas the definition of CoP states that the left-hand side of the equation is zero, the ZMP is traditionally computed from the expression that the right-hand side is zero.

Since CoP = ZMP, the ZMP may never leave the support polygon, contrary to what was incorrectly implied earlier (Li, Takanishi, and Kato 1993; Shih 1996). Also, the ZMP has no inherent relationship with a dynamically stable gait as has been previously stated (Li, Takanishi, and Kato 1993; Shih et al. 1990).

3.4. The FRI Point and the CoP

To relate the FRI point and the CoP, let us rewrite eq. (2), this time computing the moments at F:

$$\begin{aligned} \boldsymbol{M} + \boldsymbol{FP} \times \boldsymbol{R} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{g} - \boldsymbol{\tau}_1 \\ - \boldsymbol{FO}_1 \times \boldsymbol{R}_1 = \dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1. \end{aligned} \tag{16}$$

By substituting eq. (6) into eq. (16), we obtain

$$\left(\boldsymbol{FP} \times \boldsymbol{R} \right)_t = \left(\dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1 \right)_t. \tag{17}$$

The FRI point and the CoP are coincident if $\boldsymbol{FP} = \boldsymbol{0}$; i.e., if $(\dot{\boldsymbol{H}}_{G1} + \boldsymbol{FG}_1 \times m_1 \boldsymbol{a}_1)_t = \boldsymbol{0}$. This is possible if any one of the following conditions is satisfied: (1) $\boldsymbol{a}_1 = \boldsymbol{0}$ and $\ddot{\theta}_1 = 0$, i.e., the foot is at rest or has uniform linear and angular velocities; (2) $\boldsymbol{I}_1 = \boldsymbol{0}$ and $m_1 = 0$, i.e., the foot has zero mass and inertia; or (3) $\boldsymbol{FG}_1 \parallel m_1 \boldsymbol{a}_1$ and $\boldsymbol{I}_1 = \boldsymbol{0}$.

It may be shown that for an idealized rigid foot the CoP is situated at a boundary point unless the foot is in stable equilibrium. Since the position of the CoP cannot distinguish between the marginal state of static equilibrium and a complete loss of equilibrium of the foot (in both cases it is situated at the support boundary), its utility in gait planning is limited. The FRI point, on the other hand, may exit the physical boundary of the support polygon, and it does so whenever the foot is subjected to a net rotational moment.

3.5. The CoP and the GCoM

The GCoM, represented by C in Figure 1, satisfies

$$\boldsymbol{CG} \times \sum m_i \boldsymbol{g} = \boldsymbol{0}, \tag{18}$$

where G is the center of mass of the entire robot and $\sum m_i = M$ is the total robot mass. Noting that $\boldsymbol{CG} \sum m_i = \sum \boldsymbol{CG}_i m_i$ and $\boldsymbol{CG}_i = \boldsymbol{CP} + \boldsymbol{PG}_i$, we can rewrite eq. (18) as

$$\boldsymbol{CP} \times \sum m_i \boldsymbol{g} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{g} = \boldsymbol{0}. \tag{19}$$

Substituting in eq. (1), we get

$$\boldsymbol{M} - \boldsymbol{CP} \times \sum m_i \boldsymbol{g} = \sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{a}_i. \tag{20}$$

From above, P and C coincide if $\left(\sum \dot{\boldsymbol{H}}_{Gi} + \sum \boldsymbol{PG}_i \times m_i \boldsymbol{a}_i \right)_t = \boldsymbol{0}$, which is possible if the robot is stationary or has uniform linear and angular velocities in all the joints.

4. Simple Examples

The objective of this section is to elucidate the idea behind the FRI point by means of four simple examples, depicted in Figures 6 and 7. The examples are based on an idealized planar point-mass model of the shank (an inverted pendulum) connected through an "ankle" joint to a triangular foot.

4.1. Example 1

We consider an unactuated ankle joint, $\boldsymbol{\tau}_1 = 0, \dot{\theta}_1 \neq 0, \ddot{\theta}_1 \neq 0$, as shown in Figure 6a. From eq. (6), we have

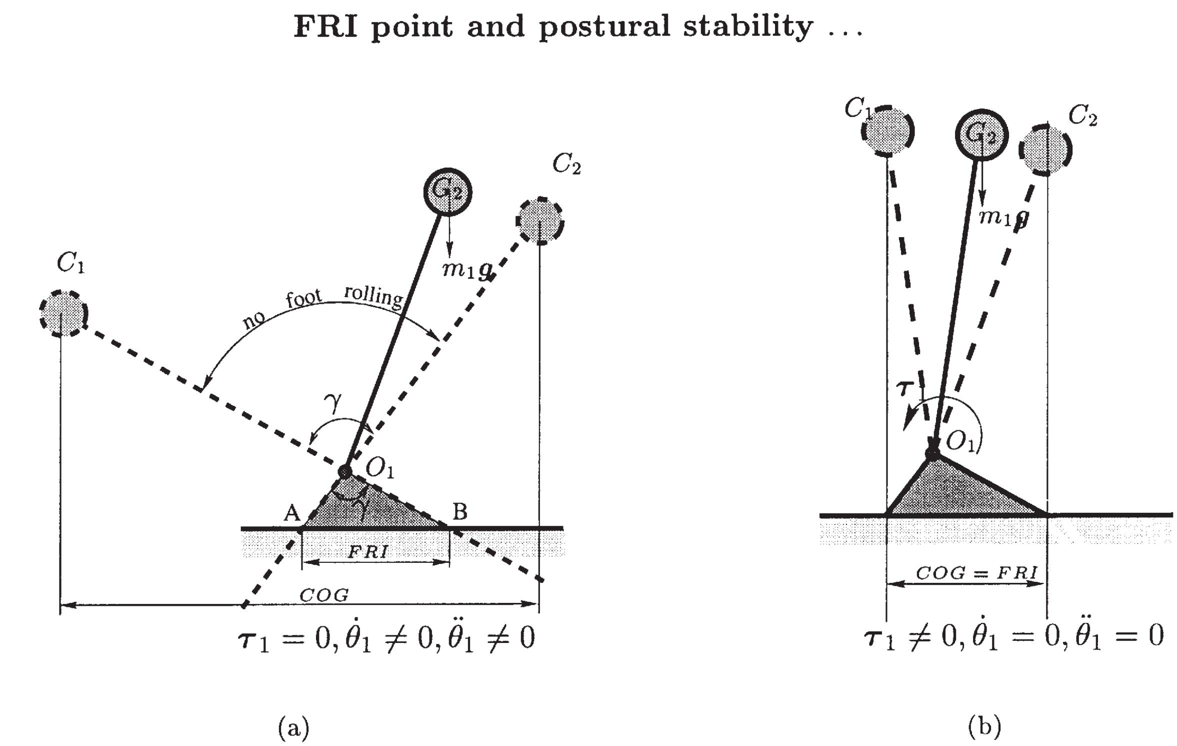

Fig. 6. Simple planar examples. The ankle joint in Example 1 is unactuated (a). The FRI point is situated on the line O_1G_1 (extrapolated) at its penetration point on the ground. In Example 2, the ankle torque is just sufficient to counterbalance the gravity moment, and the system is stationary (b). In this case, as in all other stationary mechanisms, the FRI point coincides with the GCoM and the CoP.

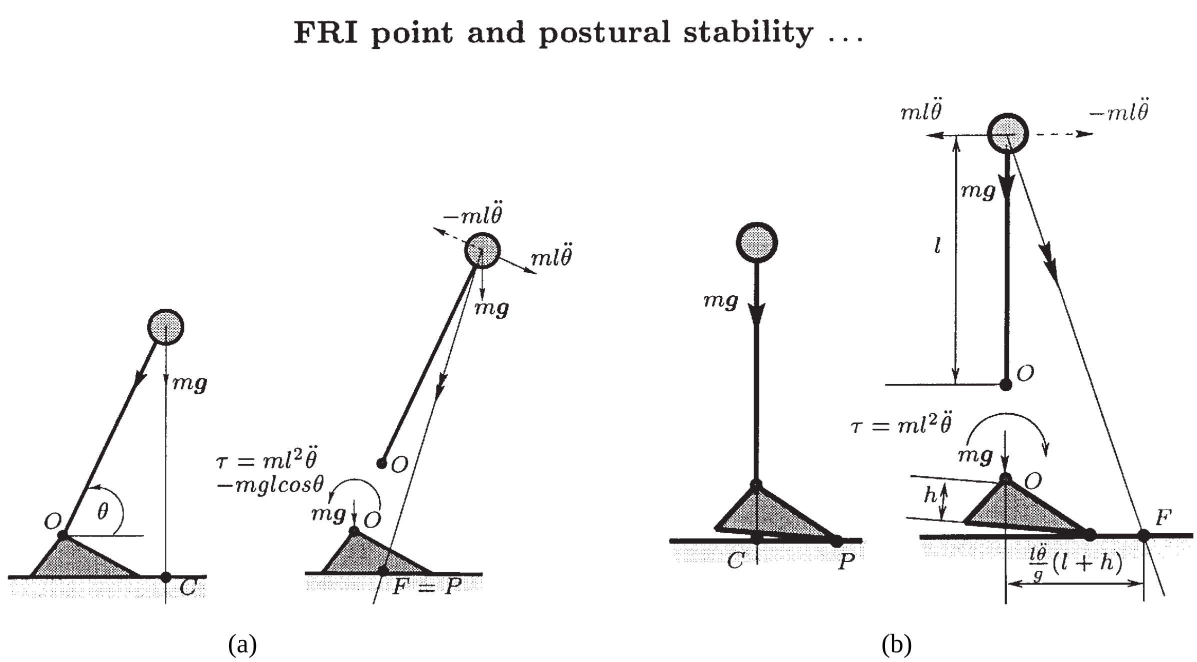

Fig. 7. Two simple examples to compare and contrast the CoP (P), GCoM (C), and FRI point (F). At left, the foot is in static equilibrium since F is within the support line (although C is outside); P is coincident with F. At right, the foot is starting to rotate, since F is outside the support line (although C is inside); P is at the tip about which the foot rotates.

$\left(\boldsymbol{FO}_1 \times \boldsymbol{R}_1\right)_t = \mathbf{0}$, assuming that $m_1 \approx 0$. For a frictionless ankle joint, $\boldsymbol{R}_1$ is always directed toward $\boldsymbol{O}_1\boldsymbol{G}_2$; in other words, if we simply extend the line $\boldsymbol{O}_1\boldsymbol{G}_2$, the point where it penetrates the ground is the position of the FRI point. Two extreme shank configurations beyond which foot rotation occurs are shown as C_1 and C_2 in the figure.

If we release the shank from a position slightly off from its vertical configuration, it will fall due to gravity while rotating around O_1. If the shank rotates clockwise, the foot will remain stable until the shank arrives at configuration C_2, at which point the foot starts rotating counterclockwise about A. On the other hand, for counterclockwise rotation of the shank, the foot starts rotating clockwise around B once the shank crosses the configuration C_1. Although the opposite rotations of the shank and the foot may appear counterintuitive at first, it is better understood by recalling that the forces acting on the two segments at the ankle joint O_1 are equal and opposite.

4.2. Example 2

Next we consider an actuated system (Fig. 6b) with an ankle torque that precisely compensates for the gravitational moment but does not generate any shank motion; i.e., $\dot{\theta}_1 = 0$ and $\ddot{\theta}_1 = 0$. To determine the position of the FRI point of this system, we use $\boldsymbol{\tau}_1 = -\boldsymbol{O}_1\boldsymbol{G}_2 \times m_2\boldsymbol{g}$ and $\boldsymbol{R}_1 = -m_2\boldsymbol{g}$ in eq. (6). We get $\sum \boldsymbol{FG}_i \times m_i\boldsymbol{g} = 0$. This means that F falls on the CG gravity line of the system. This property is valid not only for the foot/shank, but for any stationary mechanism (Shih et al. 1990).

4.3. Example 3

In the next example, shown in Figure 7 (left), the shank configuration corresponds to a GCoM position C outside the support polygon. The foot is, however, prevented from rotating by the ankle torque $(ml^2\ddot{\theta} - mg\cos\theta)$. This should be taken into consideration while planning the gait initiation of biped robots. It is noteworthy that to stop the robot from tipping over, some control laws accelerate the heavy robot body forward (Hirai et al. 1998). This generates a supplementary backward inertia force—similar to this example—which shifts the FRI point F backward, bringing it within the support polygon. Since the foot is stationary, $F = P$.

4.4. Example 4

Finally in Figure 7 (right), the shank is vertically upright with its GCoM well within the support line. Despite this, the foot starts to rotate due to the ankle torque $ml^2\ddot{\theta}$. The FRI point F is situated outside the support line at a horizontal distance $OF_y = \frac{l\ddot{\theta}}{g}(l + h)$ from O. The CoP is at the extreme frontal point of the support polygon.

5. Control Issues

Although the focus of this work is the dynamics of biped robots and the introduction of the FRI point, it is the control of this point which is of importance to the robotics community. The control issues faced are similar to those involving the control of the CoP (or ZMP), and we briefly describe the available approaches. Readers interested in the actual implementation of the control of CoP are directed to various earlier works (Vukobratovic, Frank, and Juricic 1970; Vukobratovic 1973; Takanishi, et al. 1985; Takanishi, et al. 1990; Li, Takanishi, and Kato 1992, 1993; Vukobratovic and Timcenko 1996; Shih 1996; Fujimoto and Kawamura 1996; Hirai et al. 1998).

Any control strategy for the FRI point needs to be aware of two important characteristics of legged robots: underactuation and unilaterality. Additionally, the FRI-point control falls in the category of redundant control. The ground coordinates of the FRI point are the only two independent parameters to be controlled, whereas the control input is higher-dimensional, and is equal to the number of actuated degrees of freedom of the robot. One therefore needs to impose extra constraints or other task criteria for a successful redundancy resolution.

The condition that the FRI point (and the CoP) may not exit the support polygon during a static walk is not by itself sufficient for a trajectory-tracking implementation. One of the fundamental difficulties is our inability to specify a reasonable trackable trajectory. For biped robots with human dimensions, one approach will be to track the CoP trajectory measured from human locomotion. The connection between the desired features of a locomotion and the CoP trajectory also needs to be established.

Peripherally related to the issue of control is the lack of an accepted definition of gait stability. Although static stability has a precise meaning, dynamic stability of gait seems to simply imply a lack of static stability and an indefinitely sustained gait. We have discussed elsewhere (Goswami, Thuilot, and Espiau 1998) the difficulties in appropriately defining the stability as applied to biped locomotion. One definition of stability that reflects the repetitive pattern of gait is that of the orbital stability (Hayashi 1985). Three other definitions of biped robot stability are discussed by Vukobratovic, Frank, and Juricic (1970). These are body stability, body-path stability, and stationary-gait stability. Body stability essentially implies that the body-attitude angles remain in a bounded region in the space spanned by the angles, and returns to it after a perturbation. Body-path stability guarantees that the biped-robot body returns to its original average velocity after a perturbation. Finally, the stationary-gait stability implies that the characteristic features of a gait, represented by a parameter vector, remain within a volume in the parameter space. Whereas these definitions are of obvious practical value, a mathematically more rigorous definition will be welcome.

6. Conclusions and Discussion

We have introduced a new criterion called the FRI point that indicates the state of postural stability of a biped robot. The FRI point is a point on the foot/ground surface, within or outside the support polygon, where the net ground-reaction force would have to act to keep the foot stationary. When the entire robot is stationary and stable, the FRI point is situated within the support polygon, and is coincident with the GCoM and the CoP. For stationary and unstable configurations, both the GCoM and the FRI point, which are coincident, are outside the support polygon. The CoP is at the polygon boundary.

In the presence of dynamics, the GCoM and the FRI point are noncoincident. When the foot is stable (implying that the robot possesses postural balance), the FRI point is situated within the support polygon and is coincident with the CoP. An exit of the FRI point from the support polygon signals postural instability. The CoP may never leave the support polygon. The farther away the FRI point is from the support boundary, the larger is the unbalanced moment on the foot, and the greater is the instability. The distance between the FRI point and the nearest point on the polygon boundary is a useful indicator of the static stability margin of the foot.

Although postural stability of a biped robot (or a human being) is closely related to the static stability of its foot, the relationship between foot stability and natural anthropomorphic bipedalism is not at all clear. Even a simple observation of human locomotion will convince us that a significant part of the gait cycle involves foot rotation. One of our future goals is to measure the FRI-point trajectory for natural human locomotion.

We have investigated the fundamentals of the CoP and the ZMP in this paper. Since its introduction about 30 years ago, the ZMP has found frequent mention in the robotics literature, but unfortunately, confusion about its physical nature has persisted. Some of this confusion is due to a nonrigorous choice of terms in the existing definitions. This paper lists some of the definitions that are clear and consistent. We have three major comments about this issue. First, we have shown that the CoP and the ZMP are physically identical. Second, all three moment components are not necessarily zero at the ZMP. This raises a question about the appropriateness of its name, especially in view of the first point. Third, the ZMP (being identical to the CoP) may never leave the support polygon, despite several indications to the contrary in the literature.

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References

Arakawa, T., and Fukuda, T. 1997 (Albuquerque, NM). Natural motion generation of a biped locomotion robot using the hierarchical trajectory generation method consisting of GA, EP layers. *Proc. of the IEEE Conf. on Robot. and Automat.*, vol. 1. Washington, DC: IEEE, pp. 211–216.

Banach, S. 1951. *Mechanics.* Warszawa, Poland: Monografie Matematyczne. (Trans. by E. J. Scott.)

Coussi, O., and Bessonet, G. 1995. ZMP et centre de pression (in French). unpublished.

Espiau, B. 1998. Center of pressure and the zero-moment point. Unpublished.

Fujimoto, Y., and Kawamura, A. 1996 (Minneapolis, MN). Proposal of biped walking control based on robust hybrid position/force control. *Proc. of the IEEE Conf. on Robot. and Automat.*, vol. 3. Washington, DC: IEEE, pp. 2724–2740.

Goswami, A., Thuilot, B., and Espiau, B. 1998. A study of the passive gait of a compass-like biped robot: Symmetry and chaos. *Intl. J. Robot. Res.* 17(12).

Hayashi, C. 1985. *Nonlinear Oscillations in Physical Systems.* Princeton, NJ: Princeton University Press.

Hemami, H., and Golliday, C. L. 1977. The inverted pendulum and biped stability. *Math. Biosci.* 34(1–2):95–110.

Hirai, K., Hirose, M., Haikawa, Y., and Takenaka, T. 1998 (Leuvin, Belgium). The development of the Honda humanoid robot. *Proc. of the IEEE Conf. on Robot. and Automat.* Washington, DC: IEEE.

Lee, W., and Raibert, M. 1991 (Sacramento, CA). Control of hoof rolling in an articulated leg. *Proc. of the IEEE Conf. on Robot. and Automat.* Los Alamitos, CA: IEEE.

Li, Q., Takanishi, A., and Kato, I. 1992. Learning control of compensative trunk motion for a biped walking robot based on ZMP stability criterion. *Proc. of the IEEE/RSJ Intl. Conf. on Intell. Robot. and Sys.* Los Alamitos, CA: IEEE.

Li, Q., Takanishi, A., and Kato, I. 1993 . Learning control for a biped walking robot with a trunk. *Proc. of the IEEE/RSJ Intl. Conf. on Intell. Robot. and Sys.* Los Alamitos, CA: IEEE, pp. 1771–1777.

Patla, A., Frank, J., and Winter, D. 1990. Assessment of balance control in the elderly: Major issues. *Canadian Physiotherapy* 42:89–97.

Shih, C.-L. 1996. The dynamics and control of a biped walking robot with seven degrees of freedom. *ASME J. Dyn. Sys. Meas. Control* 118(Dec.):683–690.

Shih, C.-L., Li, Y. Z., Churng, S., Lee, T. T., and Gruver, W. A. 1990. Trajectory synthesis and physical admissibility for a biped robot during the single-support phase. *Proc. of the IEEE/RSJ Intl. Conf. on Intell. Robot. and Sys. (IROS)*. Los Alamitos, CA: IEEE, pp. 1646–1652.

Takanishi, A., Ishida, M., Yamazaki, Y., and Kato, I. 1985. The realization of dynamic walking by the biped robot WL-10RD. *Proc. of the Intl. Conf. on Adv. Robot.* Tokyo, pp. 459–466.

Takanishi, A., Takeya, T., Karaki, H., and Kato, I. 1990. A control method for dynamic biped walking under unknown external force. *Proc. of the IEEE/RSJ Intl. Conf. on Intell. Robot. and Sys. (IROS)*. Los Alamitos, CA: IEEE, pp. 795–801.

Vukobratovic, M. 1973. How to control artificial anthropomorphic systems. *IEEE Trans. Sys. Man Cybernet.* 3(5):497–507.

Vukobratovic, M., Borovac, B., Surla, D., and Stokic, D. 1990. *Scientific Fundamentals of Robotics 7. Biped Locomotion: Dynamics Stability, Control, and Application*. New York: Springer-Verlag.

Vukobratovic, M., Frank, A. A., and Juricic, D. 1970. On the stability of biped locomotion. *IEEE Trans. Biomed. Eng.* 17(1):25–36.

Vukobratovic, M., and Juricic, D. 1969. Contributions to the synthesis of biped gait. *IEEE Trans. Biomed. Eng.* 16:1–6.

Vukobratovic, M., and Timcenko, O. 1996. Experiments with nontraditional hybrid control of biped locomotion robots. *J. Intell. Robot. Sys.* 16:25–43.

Winter, D., Ruder, G. D., and MacKinnon, C. D. 1990. Control of balance of upper body during gait. In Winters, J. M., and Woo, S. L.-Y. (eds.) *Multiple Muscle Systems: Biomechanics and Movement Organization*. New York: Springer-Verlag, pp. 534–541.